Black-hole thermodynamics

Jacob D. Bekenstein

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Including black holes in the scheme of thermodynamics has disclosed a deep-seated connection between gravitation, heat and the quantum that may lead us to a synthesis of the corresponding branches of physics.

Jacob D. Bekenstein

To the physicist casually interested in gravitation, a black hole is a passive object that swallows anything near it and cannot be made to disgorge it; it absorbs but cannot emit. At the close of the last decade the experts shared this view. Recently, however, this simple picture has changed entirely. Perhaps no single development highlighted more the new views about black holes than the quantum argument presented by Stephen Hawking of Cambridge University in 1974 that a black hole must radiate spontaneously with a thermal spectrum. The importance of this phenomenon is not so much in possible practical applications, not even in its astrophysical implications, but rather in that it has confirmed earlier suspicions that gravitation, thermodynamics and the quantum world are deeply interconnected. This connection, which might be symbolized by the thermodynamic engine shown in figure 1, engenders hope that we may achieve a synthesis of these three branches of physics in our time and bears witness to the profound unity of physics, a unity too often veiled in an age of increasing specialization.

Black holes emerged as solutions of the gravitational field equations of Albert Einstein's general relativity which describe regions of space-time invisible from their exterior. The first such known solution, found in 1916 by the noted physicist and astronomer Karl Schwarzschild, represents the space-time geometry (gravitational field) of a spherical static black hole; the only adjustable parameter of the solution is the object's mass M. In a famous paper, published on the day marking the outbreak of World War II, J. Robert Oppenheimer and Harlan Snyder demonstrated that this Schwarzschild solution describes the final state of a

Jacob D. Bekenstein is Professor of Physics at the Ben-Gurion University of the Negev, Beer-Sheva, Israel.

spherically collapsing massive star (see figure 2). This paper was the first to regard the black hole as a central phenomenon in astrophysics. But this is not the aspect of interest here. Rather, we are interested in the black hole as a state of the gravitational field that resembles an ordinary object in many respects, especially in its interactions with the rest of the universe.

By the 1960's the most general black hole solution known was one describing a rotating electrically charged hole in stationary state. It is parameterized by the object's mass M, charge Q and angular momentum L (see table 1), and is generally known as the charged Kerr black hole[1] after Roy Kerr who discovered the special case with $Q = 0$ in 1962. The Schwarzschild hole has $L = Q = 0$.

Black holes have no hair

In principle one would expect equilibrium black holes with more parameters describing shape and various other properties. Yet in the late 1960's John A. Wheeler, then at Princeton University, suggested[1] that in fact the charged Kerr black holes are the most general equilibrium black holes states as far as exterior properties are concerned. (See also the article by Remo Ruffini and Wheeler, "Introducing the black hole," PHYSICS TODAY, January 1971, page 30.) He was led to this conjecture, which he whimsically paraphrased as "black holes have no hair," by uniqueness theorems of Werner Israel and Brandon Carter regarding the Schwarzschild and Kerr holes. Since then an impressive amount of evidence has piled up in favor of the conjecture; in particular, it has become clear that there is no way to introduce quantities like baryon and lepton numbers, strangeness, etc. as black-hole parameters. True, over the years black-hole solutions having magnetic monopole, scalar

field charge, quark color, and other denizens of the theorist's mind as parameters have been exhibited. But no extension of the charged Kerr solution has ever emerged as a description of equilibrium black holes, and no "quantum number" other than M, Q, and L, has been found to characterize the state of black holes. (For example, its baryon and lepton numbers would be unobservable outside the black hole, the magnetic pole strength is, as far as we know, not a freely occurring quantity and present theory regards all observable hadrons—from which the black hole would form—as "colorless.")

This conclusion appears to be theory independent. Black-hole solutions of several of general relativity's competitor gravitational theories (scalar–tensor theories and supergravity) have been found. They all belong to the charged Kerr family. The principle "black holes have no hair" evidently transcends the bounds of general relativity, and may be regarded as a general law of black-hole physics.

Irreversibility of black holes

As a graduate student of Wheeler's at Princeton I found "black holes have no hair" distressing for a reason he brought home to me in a 1971 conversation. The principle, he argued, allows a wicked creature—call it Wheeler's demon—to commit the perfect crime against the second law of thermodynamics. It only has to drop a package containing some entropy into a stationary black hole, thus decreasing the entropy in the part of the universe visible from the exterior. The associated changes in M, Q, and L do not uniquely reveal how much entropy is then inside the hole, so an *exterior* observer with no inside information about the package can never be sure that the total entropy in the universe has not decreased. For him the second law is transcended—

made irrelevant. It loses its predictive power, so that black holes seem to be outside the province of thermodynamics. This circumstance seemed disastrous, not only because it would deprive us of the use of model-free thermodynamic reasoning in investigating the bizarre black holes, but also because it could be seen as throwing doubt on their very existence, even in principle.

While mulling over this dilemma I was struck by the possible relevance of a discovery that Demetrios Christodoulou, another of Wheeler's students, had made not long before. While investigating the efficiency of certain processes proposed by Roger Penrose for extracting rotational energy from a Kerr black hole and converting it to mechanical energy of particles, Christodoulou noticed that the most efficient processes are those associated with reversible changes of the black hole.[1] Less efficient processes are all connected with the irreversible increase of a certain "irreducible mass" M_{ir}—the inextractable part of the mass of the hole. Since in thermodynamics reversible processes are the most efficient ones for converting energy from one form into another, there was a clear thermodynamic ring to all this. So black holes might be in harmony with thermodynamics after all.

If there were any doubts as to the generality of the irreversibility, they were dispelled by a theorem proved by Hawking: The surface area A of the boundary or horizon of any black hole cannot decrease and will increase in a dynamical process. For a Kerr hole A is proportional to M_{ir}^2, so the area theorem implies Christodoulou's result that all but very idealized processes increase M_{ir}. But the theorem shows irreversibility to be a property of all black hole processes, not just those of near-equilibrium holes. And it points to a formal analogy between black hole area and entropy of a closed system—both like to increase.

Black-hole entropy

It occurred to me that some monotonic increasing function of A might play the role of entropy of black holes—entropy on the same footing as ordinary thermal entropy. In my 1972 dissertation I discussed various questions connected with such an identification, and showed how to use it to defeat the schemes of Wheeler's demon and make black hole physics consistent with thermodynamics.

The first question was, what function of A is to be identified with the entropy S_{bh} of the black hole. There are several other conditions an entropy function must meet besides the inability to decrease spontaneously: It must also be additive for independent systems, for example. From the outset the most likely prospects were proportionalities to A or to $\sqrt{A}$. The second appeared especially attractive, because it makes the entropy proportional to the mass for black holes
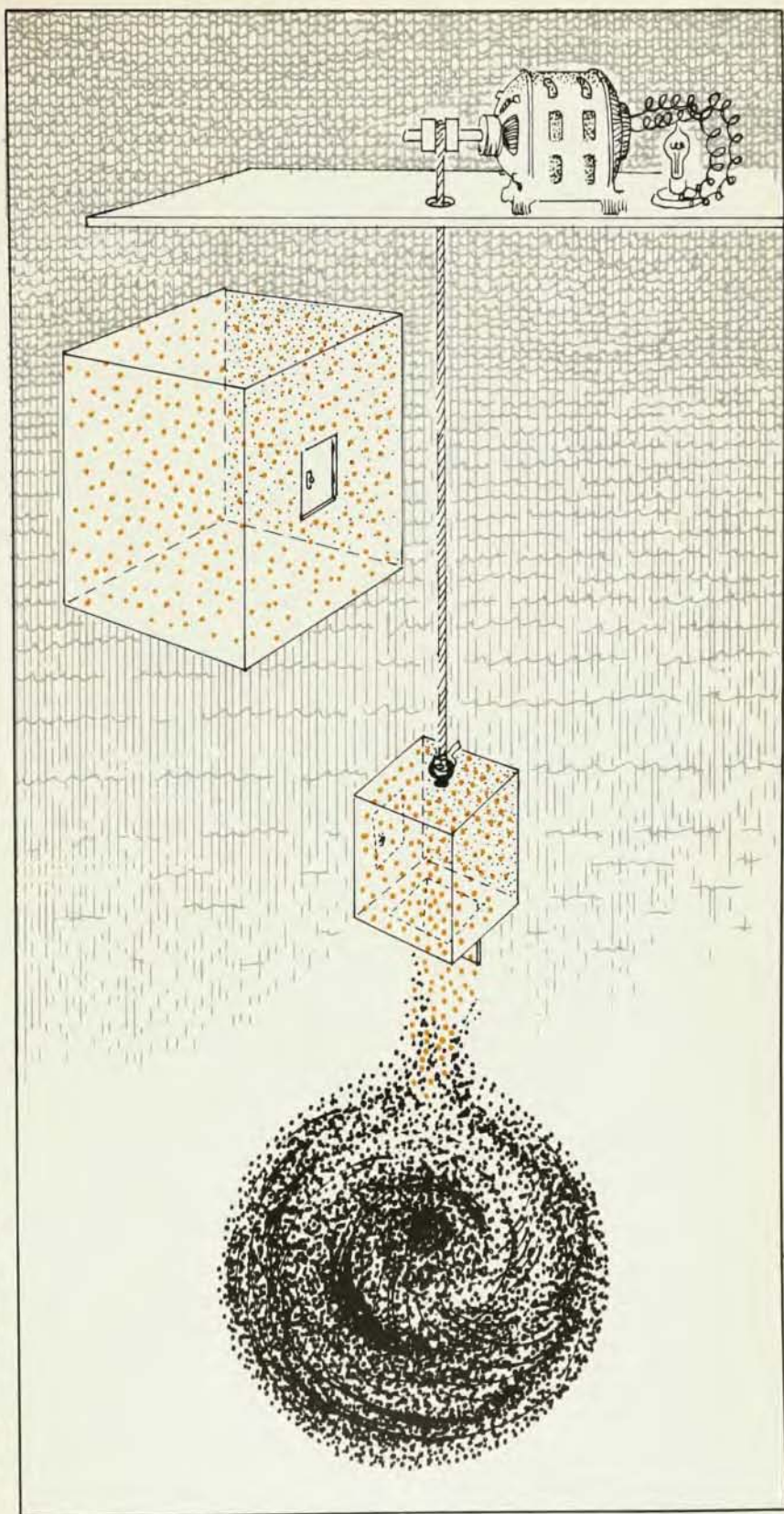
A gravitational–thermodynamic engine for converting heat to work, conceived by Robert Geroch, uses a black hole as heat sink. The box is filled with thermal radiation and lowered in the hole's field to its horizon. The gravitational field does work on the box in this process. The radiation is then allowed to escape into the hole, and the box is hauled back up at expense of less work than was obtained in the first stage. It can then be refilled with thermal radiation from the reservoir. Because the box cannot be allowed to touch the horizon, the efficiency of conversion is less than 1 and not unity as originally thought. (Drawing by Louis Fulgoni)   Figure 1
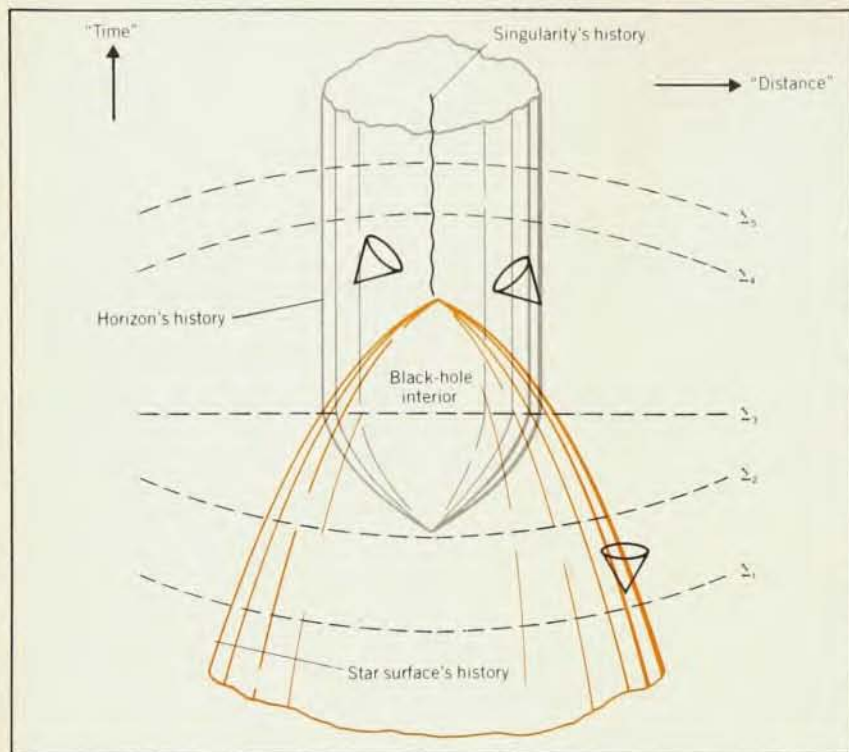
Spherical collapse to a black hole. We show the spacetime histories of the collapsing body's surface (illustrated as one-dimensional), of the resulting singularity and of the horizon expanding to engulf the body. The tilting of the light cone by gravitation explains why the region interior to the horizon is invisible from outside. The spacelike hypersurfaces labeled Σ supplant, in relativity, the concept of space at a given time. **Figure 2**

with the same ratios of L/M^2 and Q/M; for ordinary matter, also, the entropy is proportional to mass, other things being equal. But this choice leads to trouble. When two black holes coalesce, Hawking's area theorem says the surface area must be at least as large as the sum of the areas of the two holes. However, the sum of the square roots of the original areas can exceed the square root of the final area, so making S_{bh} proportional to $\sqrt{A}$ would endow it with unacceptable behavior. No problem arises if S_{bh} is proportional to A. Thus I chose[2] S_{bh} to be proportional to the area of the horizon.

Determination of the proportionality constant was more problematic. There is an obvious constant with dimensions of entropy—Boltzmann's constant k. What was missing was the scale of length whose square should be used to reduce A to a dimensionless quantity. Wheeler suggested the fundamental length

$$L_{PW} = (\hbar G/c^3)^{1/2} = 1.61 \times 10^{-33} \text{ cm}$$

which was introduced into physics by Max Planck after his discovery of the law for the blackbody spectrum. Planck noted that this scale involves fundamental constants independent of particle properties, in contrast to other lengths such as the classical radius of the electron. Later, Wheeler stressed the role L_{PW} must play in the ultimate theory of

quantum gravitation. He has argued that L_{PW} must represent the smallest scale at which spacetime can be regarded as a smooth manifold; at smaller scales it must have a foam-like consistency; L_{PW} is often called the Planck–Wheeler length.

Wheeler's suggestion led[2] to a formula for black hole entropy independent of the properties of matter

$$S_{bh} = \eta k A/L_{PW}^2$$

Here η is a pure number, presumably of order unity, to be determined separately. This formula links a purely thermodynamic quantity—entropy—to a purely gravitational one—horizon area, and the

connection becomes meaningless if one goes to the classical limit $\hbar \to 0$. Thus it reflects the deep connection of gravitation, thermodynamics and the quantum world I alluded to.

Information and entropy

By now the reader of these lines must be wondering about the physical meaning of S_{bh}. It cannot be the thermal entropy of the matter inside the hole. For consider a sun-like star with mass 2×10^{33}g. Its thermal entropy is $10^{58}k$ in order of magnitude. Were the star to collapse completely, it would make a stationary black hole with surface area of order 100 km^2 (see table 1). Our formula associates with it an entropy of order $10^{78}k$. Yet no known dissipative process can generate enough entropy during the collapse to multiply the matter's entropy by a factor 10^{20}. Thus S_{bh} is something else than thermal entropy. In statistical physics, as well as other fields, entropy has come to stand for missing information. With that interpretation, the thermal entropy of a cube of sugar simply measures our ignorance as to the precise microscopic state of the molecules in the cube, while its macroscopic state is fully described by chemical composition, temperature, volume and perhaps a few other variables. The ignorance of the *exterior* observer about the matter inside a black hole is deeper. "Black holes have no hair" allows us only knowledge of M, Q and L for a stationary hole; neither the microstate, nor composition (baryons, photons...), nor temperature, nor structure (shape, size...), nor anything else is—even in principle—measurable by a distant observer. The function S_{bh} is the obvious candidate for quantifying this deeper ignorance, so it is not surprising that it can be vastly larger than any reasonable estimate of the thermal entropy.

The following reasoning shows this interpretation of S_{bh} is not far fetched. When an object disappears down a black hole, we lose information about its microstate, composition, motion, and so on. (The information about the motion is clearly tied with properties of the hole in whose field the object moves. But clearly

Table 1 The charged Kerr black hole

| Parameter	mass M	charge Q	angular momentum L
Characteristic length	$m = GM/c^2$ roughly size of hole	$q = (GQ^2/c^4)^{1/2}$	$a = L/Mc$
Quantity equivalent to 1 cm	1.35×10^{28}g	3.49×10^{24}esu	3×10^{10} cm^2/s specific angular momentum
Constraint	$m^2 \geq q^2 + a^2$		
Formula for horizon area	$A = 4\pi\{[(m + (m^2 - q^2 - a^2)^{1/2}]^2 + a^2\}$		
Potential	surface tension θ	electric potential Φ	angular velocity Ω
Definition	$(\partial Mc^2/\partial A)_{L,Q}$	$(\partial Mc^2/\partial Q)_{L,A}$	$(\partial Mc^2/\partial L)_{Q,A}$
Formula	$(m^2 - q^2 - a^2)^{1/2}c^4/2GA$ $c^4/2G = 6.07 \times 10^{48}$ erg/cm	$Q[4\pi/A - (4\pi a/A)^2]^{1/2}$	$(L/M)4\pi/A$

the *minimum* of the total lost information taken over all possible motions depends only on properties other than the motion and should be independent of the hole's properties.) If the identification of S_{bh} as information hidden inside the black hole is correct, the minimum possible growth in S_{bh} from injection of a given body into a hole in any orbit should be independent of the hole's properties, the same for a big hole or a small one, a static or a rotating one. *A priori*, no mechanical reason is evident for why this should be so. Yet detailed calculations[2,3] showed that for injection into a stationary hole the minimum growth in S_{bh} is $8\pi\eta k\mu cr_0/\hbar$ where μ is the object's mass and r_0 its effective radius. This result is independent of the hole's parameters, as required. Thus the interpretation is consistent.

In statistical physics the entropy divided by k gives the logarithm of the number of microstates compatible with the given macrostate. It is thus reasonable to expect that $\exp(S_{bh}/k)$ gives the number of possible interior configurations of the black hole compatible with the given exterior state whose black hole entropy is S_{bh}. Here "interior configuration" refers to each possible microstate of composition and structure counted separately (see figure 3). This seemingly unverifiable relation has, in fact, very palpable consequences to which I will come later. Just now I want to show how it leads to an estimate of the constant η.

Consider a hole with entropy S_{bh} and, consequently, $\exp(S_{bh}/k)$ possible interior configurations. Inject into it the simplest of systems, a generic elementary particle. This has several possible microstates differing in spin, charge, and other quantum numbers. So the number of interior configurations after the injection will exceed $2 \exp(S_{bh}/k)$ since each old configuration together with one microstate makes a new configuration. Hence the growth in black hole entropy exceeds $k \ln 2$. But our previously mentioned result assures us that the minimum growth in S_{bh} is $8\pi\eta k$, because r_0 is $\hbar/\mu c$ for an elementary particle. Comparing the two values, we infer a lower bound (ln 2)/8π, or 0.028, for η. This was the best known estimate for η until Hawking determined it to be $\frac{1}{4}$, an order of magnitude larger.

The generalized second law

Black-hole entropy is just what is needed to solve the paradox posed by Wheeler's demon, for it makes possible a generalization of the second law: "the sum of the black-hole entropy and the ordinary thermal entropy outside black holes cannot decrease." By speaking only about quantities determinable from outside the holes, this law avoids the problem the ordinary second law ran into. It is not transcended. It makes a statement subject to verification: that the decrease in the outer world's entropy following the
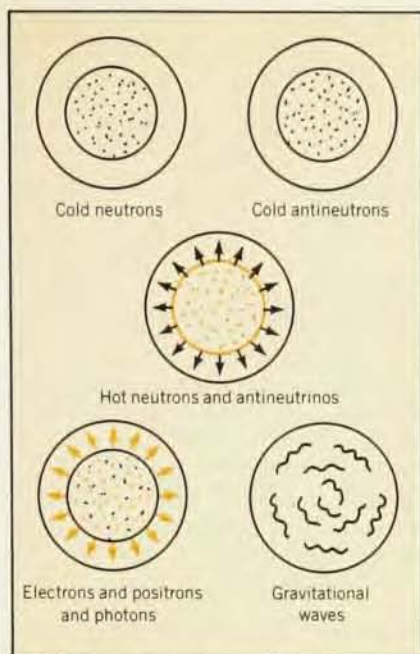
Interior configurations of a given black hole are microstates of all possible structures and compositions, a very few of which are shown. These "configurations" include states of different baryon and lepton numbers, so long as total mass, charge and angular momentum are preserved. They need not be just arrangements of matter; they can also include gravitational waves, for example, which are regarded as vacuum systems in general relativity.   Figure 3

infall of the package will be compensated, or even overcompensated, by an increase in S_{bh}.

In 1972 when I conjectured this generalized second law, it was not clear that it would always work. In fact, nearly everybody I discussed it with objected that it must often fail because the growth in black-hole area is caused by the mechanical properties of the infalling body, which are unrelated to its content. In my dissertation, and in later work when at the University of Texas at Austin, I countered this objection by using the result about the minimum increase in S_{bh} and by calculating the entropy content for various simple systems.[2,3] The calculations suggested that there is a limit on the entropy that can be packed in a body of given mass and size. In every case checked in which a small package of entropy falls into a stationary hole, the generalized second law was found to work (see figure 4). It could thus be taken as the second law of a black-hole thermodynamics.

Of course, a law is not proved by specific instances; it cannot be proved. Rather it gains credibility with each example in which it works. If, in addition, the proposed law passes the test posed by a novel or unfamiliar situation, one assumes that it describes nature correctly. We shall see that the generalized second law passed just such a test after it seemed

that there were situations it could not handle.

Black hole temperature

Consider the expression for S_{bh} of a charged Kerr hole (see table 1); its total differential with respect to M, Q and L can be written as[2]

$$T_{bh}dS_{bh} = d(Mc^2) - \Phi dQ - \Omega dL$$

where T_{bh} is defined in terms of the nonnegative quantity θ of table 1 by $T_{bh} = L_{PW}^2\theta/\eta k$, and Φ and Ω turn out to be the conventionally defined electric potential and rotational angular frequency of the hole's surface (see table 1), so that ΦdQ is the work done on the hole by adding to it charge dQ, and ΩdL is the work done by addition of angular momentum dL. Of course, $d(Mc^2)$ is the corresponding change in the hole's energy. Thus the expression above has exactly the form of the usual expression for the first law of thermodynamics (combined with the definition of entropy) for an ordinary equilibrium system at temperature T_{bh}. The expression invites us to regard S_{bh} as a genuine entropy if we are willing to regard T_{bh} as a quantum temperature for an equilibrium black hole—quantum because $\hbar$ appears in it. For the numerical value of T_{bh} as function of black hole mass, see figure 5. Note the curious fact that the smaller Mc^2, the higher T_{bh}. Ordinarily the lower the energy content of a body, the colder it is, other things being equal.

The physical meaning of T_{bh} proved elusive. One clearly cannot measure a temperature by sticking a thermometer into a black hole; already when one brings it into the neighborhood of the hole, the instrument would be torn to pieces by tidal forces. My favorite argument for bringing out the physics in T_{bh} revolved about a heat engine conceived by Robert Geroch, illustrated in figure 1, that employs a black hole as a heat sink. A temperature of order T_{bh} enters into the expression for the efficiency of this engine in the same way the heat sink's temperature enters the efficiency of a Carnot engine in a down-to-earth situation.[3] But the argument could not nail down the precise value of the hole's effective temperature. Dennis Sciama of Oxford University has pointed out that this deficiency and some subtle conceptual problems in the discussion were a premonition of the existence of Hawking's radiation. At that time, though, the argument was important in invalidating Geroch's claim that the process could violate the Kelvin statement of the second law. It does not, and it also respects our statement of the generalized second law.

Regarding T_{bh} as a black-hole temperature created a severe problem for thermodynamics of black holes. Suppose a black hole is immersed in thermal radiation of temperature T, with T less than
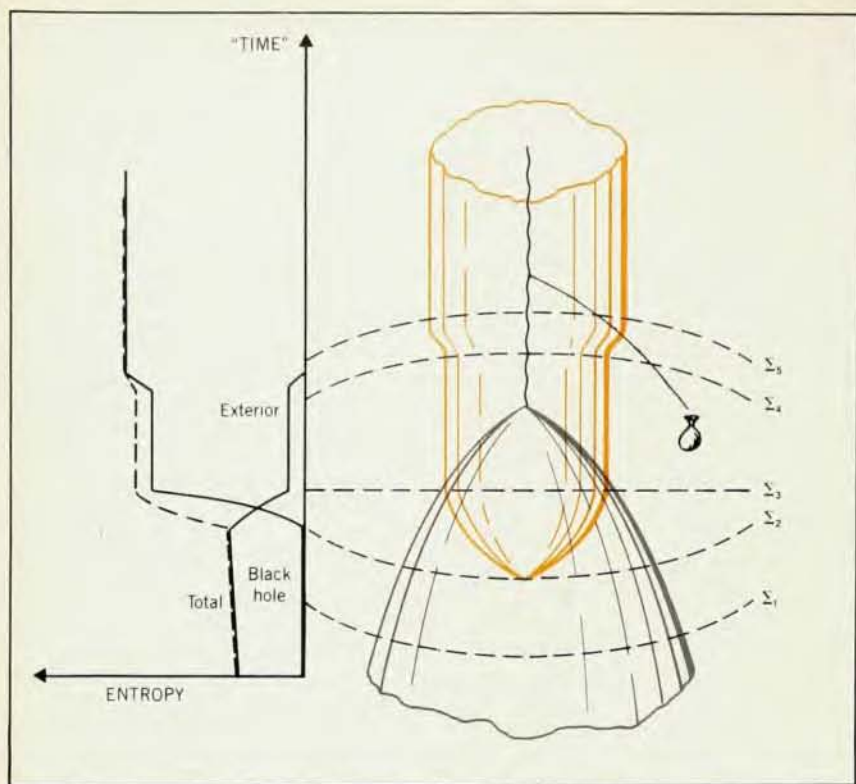
The generalized second law supplants the ordinary second law, which it transcends. The entropy on the visible parts of the hypersurfaces decreases from Σ_2 to Σ_3 as the horizon engulfs the body and from Σ_4 to Σ_5 as the entropy package thrown by Wheeler's demon falls in. However, the black-hole entropy S_{bh} increases during these intervals and the sum of the two entropies grows monotonically, as shown in the accompanying graph. **Figure 4**

T_{bh}. A simple estimate shows the hole to absorb more thermal entropy by simply sucking in radiation than the growth in black hole entropy can compensate for: there is an apparent flagrant violation of the generalized second law. My attempts to clear up this paradox were forced and inelegant.[3] The same problem does not arise for an ordinary hot body in a colder radiation bath because the body also radiates and generates thermal entropy. Classically a black hole cannot radiate, so this observation would seem irrelevant. However, the ubiquitous appearance of $\hbar$ in T_{bh}, S_{bh} and the generalized second law shows that one is here dealing with a quantum issue. The possibility that a black hole could radiate by virtue of its quantum temperature was thus apparent very early. Yet the rudimentary understanding of quantum processes in a black hole's vicinity available in 1973 gave no evidence for or against such radiation. Thus I chose to try to resolve the paradox by more conventional means[3] and in so doing lost the chance to make a striking prediction with black-hole thermodynamics. Be that as it may, the paradox was soon to find its resolution.

Black hole mechanics

This was the state of the art in mid-1973. Black-hole thermodynamics, summarized in table 1, was received with nearly universal skepticism. Most people inclined towards the viewpoint formulated by James Bardeen, Carter and Hawking at the Les Houches 1972 summer school on black holes, and later summarized in a lucid paper, "The Four Laws of Black Hole Mechanics."[4] They regarded the analogy between black holes and thermodynamics as suggestive but purely formal, reflecting no profound kinship of the two subjects, and being unconnected with the quantum. They recognized θ as the analog of temperature, but regarded the thermodynamic temperature of a black hole as zero, because apparently a black hole cannot be in equilibrium with thermal radiation at finite temperature. They regarded the horizon area A as an analog of entropy, but thought the thermodynamic entropy of a black hole to be infinite. The role of second law they gave to the area theorem, while rejecting the generalized second law as invalid, believing (erroneously, as is clear today) that it is possible to add entropy to a black hole without incrementing its area. Table 2 compares the tenets of this "black-hole mechanics" with those of black-hole thermodynamics.

There can be no question that black-hole mechanics was, at the time, the "common-sense" approach, and seemed to stand on solid ground. By contrast, black-hole thermodynamics was frankly speculative. What induced me to stick with it was the prospect of employing the rich arsenal of thermodynamics and statistical physics to attack the issue of the quantum aspects of black holes, a prospect nonexistent in black-hole mechanics. In those days in 1973 when I was often told that I was headed the wrong way, I drew some comfort from Wheeler's opinion that "black-hole thermodynamics is crazy, perhaps crazy enough to work."

Black-hole radiance

Early in 1974 Hawking announced his discovery of the thermal radiance of black holes.[5] He had been studying the theory of quantum phenomena in the neighborhood of a microscopic black hole; years earlier he had proposed that such objects may have been formed in profusion in the early cosmos. By applying the technique of second quantization to a boson field (such as the electromagnetic field) evolving in the vicinity of a collapsing spherical object, he found that the collapse creates quanta of the field that escape to infinity. As the collapse winds up with the formation of a Schwarzschild black hole, the emission, instead of dying out as might be naively expected, attains a form and rate independent of details of the collapse. Its spectrum is thermal: the mean number of quanta emitted in one mode of frequency ω is given by

$$\bar{n} = \Gamma[\exp(\hbar\omega/kT) - 1]^{-1}$$

where Γ is the hole's absorptivity (fraction of incident waves absorbed) for classical waves in the given mode, and T is a certain characteristic temperature. (A similar result holds for fermions.) The emissivity of a black hole has the same Planck distribution required by Kirchhoff's laws for the emissivity of any hot body of absorptivity Γ and temperature T. Hawking thus found that the hole radiates like a hot, non-black ("gray") body. The luminosity is proportional to M^{-2} and exceeds 1 watt for $M \lesssim 6 \times 10^{19}$g (corresponding to a size less than 10^{-8} centimeter).

Hawking also gave an argument showing that charge and rotation do not alter the thermal nature of the emission.[5] They only affect the form of the characteristic temperature T. In every case it is given by the same formula as T_{bh} with the choice $\eta = \frac{1}{4}$.

Hawking's discovery surprised everybody; of all I was probably the most pleased for it provided the missing pieces of black-hole thermodynamics. The result verified the contention that a nonzero quantum temperature is associated with a stationary black hole; it left no doubt that the Hawking temperature is identical to T_{bh}; it fixed the value of the elusive η; and it revealed the physical meaning of T_{bh}: it is the temperature of the quantum radiation from the black hole.

Hawking realized that his discovery solved the paradox posed by the black

hole in a colder radiation bath. He gave a simple proof[6] that after the radiation emitted by the hole is assimilated into the ambient bath, the sum of S_{bh} and exterior radiation entropy is larger than before. Thus the generalized second law does in fact hold because the hole generates sufficient thermal entropy. (The moment-by-moment applicability of the law has been established by an explicit statistical calculation of the radiation entropy.[7]) Having started as a vocal critic of the generalized second law, Hawking became the person who made it fully consistent with the gedanken experiments.

Actually, an isolated radiating hole illustrates the power of the generalized second law even more strikingly. The radiation causes the hole's surface area to decrease steadily—a flagrant (quantum) violation of the second law of black hole mechanics (a classical theorem). However, a detailed statistical calculation[7] shows that the increase in exterior (radiation) entropy exceeds the decrease in S_{bh}, so the generalized second law in its original formulation is obeyed (see figure 6). The Hawking radiation process thus provided a novel and unexpected test, which the (classical) area theorem failed, but which the (quantum) second law passed successfully. Ironically the area theorem, one of the motives for the introduction of black-hole entropy, has fallen victim of the revolution in our understanding of black holes. Its replacement is a new law of apparently wide applicability, one joining together hitherto distinct aspects of nature—gravitation and heat, gravitation and the quantum.

Radiance and superradiance

The theoretical necessity for Hawking's emission has been verified by many workers and by varied approaches. All the explanations of the phenomenon are technically complicated. In seeking a physical intuition of the process it pays to first inquire how it differs from the other known type of black-hole radiance. In 1971 the versatile physicist and astrophysicist Yakov B. Zel'dovich of the Soviet Academy of Sciences conjectured[8] that a Kerr hole should emit bosons spontaneously in those modes whose angular frequency ω and "magnetic" quantum number m satisfy the condition

$$\omega < m\Omega$$

Zel'dovich, and independently Charles Misner, had pointed out that classically the hole will amplify radiation scattered off it in these modes, which ever since then have been known as superrradiant modes. It was natural to regard this classical "superradiance" as a manifestation of quantum stimulated emission, and Zel'dovich inferred that it must be accompanied by spontaneous emission as in other contexts in physics. His conjecture was verified by William Unruh, then at the University of California at Berke-
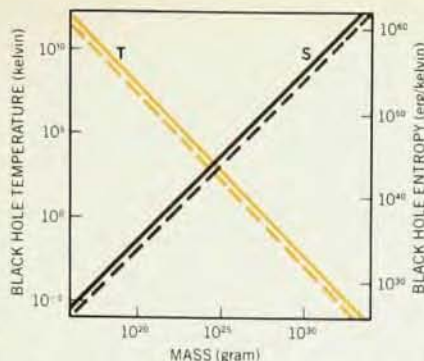
Black-hole temperature (color) and black-hole entropy (black) as functions of black hole mass ($\eta = \frac{1}{4}$). The solid lines are appropriate to a Schwarzschild hole, that is, for $q^2 + a^2 = 0$; broken lines apply to a rotating or charged Kerr hole with $q^2 + a^2 = 0.99 \, m^2$ (this is nearly the largest possible charge or spin a black hole can have—see table 1). Figure 5

ley, just before Hawking's discovery. Unruh studied a second-quantized scalar field (a meson field) evolving in the background gravitational field of an eternal Kerr black hole. He found[9] an outflux of energy in the superradiant modes for a quantum state containing no quanta incident on the hole.

In the Zel'dovich–Unruh radiance the energy comes from the hole's rotation; there is no radiance for a Schwarzschild hole. The area theorem is respected. By contrast Hawking's radiance occurs for Schwarzschild as well as Kerr holes. The energy must evidently be drawn from the "irreducible" mass (equal to the mass for Schwarzschild holes) by virtue of the quantum violation of the area theorem mentioned earlier. The Hawking radiation emerges in every mode, and it is distinguished from the first type by its thermal nature. And it occurs only if the hole was formed by collapse while the Zel'dovich–Unruh emission comes only from a hole that has existed since the infinite past.

The role of the horizon

Hawking's heuristic picture[5,10] of the process he deduced by a complicated mathematical argument is disarmingly simple. We know that in the absence of exterior fields the vacuum state of any

physical field is characterized by a multitude of virtual particle–antiparticle pairs continually appearing and annihilating each other. The pairs cannot endure; the creation of one out of nothing entails a violation of the conservation of energy, so it must disappear before the time allotted to it by the uncertainty principle runs out. But if a pair is created near a black-hole horizon, the story can be different. Hawking points out that one of the members of the pair may have time to tunnel through the horizon into one of the negative-energy particle orbits or states that exist inside the hole due to the colossal binding effects of gravitation. Its companion is left with positive energy and may be able to escape from the black hole's pull as a free, real particle. In the final analysis, its energy comes from the hole, so the process is effectively one of radiation by the hole. No overall violation of the conservation of energy occurs; the ingoing particle carries negative energy into the hole, whose mass is thereby decreased. This flux of negative energy through the horizon is what causes the breakdown of the area theorem.

The pair created near the horizon is in what one calls a pure quantum state (described by a wave function), the farthest thing in the world from a thermal state, which is a mixed state (described by a density matrix) of maximal entropy. But as Sciama has observed in another context, the disappearance of the particle (or antiparticle) behind the horizon breaks the correlations inherent in the pure state, leaving a mixed state. The loss of information into the hole is complete, so the entropy of the new state is maximal. Thus one understands the thermal nature of the radiation. By the same token it is clear why the Zel'dovich–Unruh radiation is not thermal. Its origin parallels that of the Hawking radiance. But the relevant negative energy states are now those in the ergosphere of a Kerr hole—the region girding the horizon. (These states are the ones that make the Penrose energy-extraction processes possible.) Because the trapped member of the pair does not have to cross the horizon, no great loss of information ensues, and we don't get a thermal state. For this reason also there is no violation of the area theorem.

The alert reader will have noticed the

Table 2   Black-hole mechanics versus black-hole thermodynamics

Concept	Black-hole mechanics	Black-hole thermodynamics
entropy	A is like entropy; the physical entropy is infinite	The black-hole entropy S_{bh} is $\eta c^3 kA / Gh$
temperature	θ is like temperature; the physical temperature is zero	The black-hole temperature T_{bh} is $Gh/\eta c^3 k$
first law	$\theta \, dA = dMc^2 - \Phi dQ - \Omega dL$	$T_{bh} dS_{bh} = dMc^2 - \Phi dQ - \Omega dL$
second law	For one black hole A cannot decrease; when black holes coalesce the total horizon area increases (area theorem)	The sum of S_{bh} and the entropy exterior to black holes cannot decrease (generalized second law)
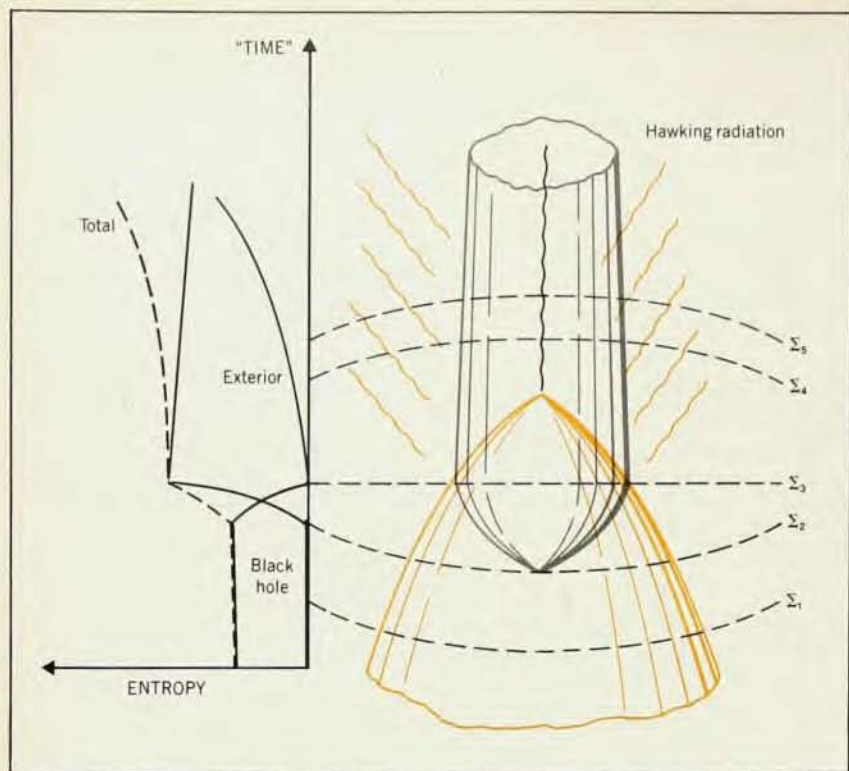
The Hawking thermal radiation begins emerging when most of the collapsing body is inside the horizon. The associated flow of negative energy through the horizon causes a steady decrease of S_{bh} from Σ_3 on. However, the growing entropy of the radiation is sufficient to make the sum of the entropies inside and outside the hole increase, as shown in the graph.   **Figure 6**

crucial role of the horizon. Whichever way we look at it, by black-hole thermodynamics or from the field-particle point of view, it is the horizon that, by its information-hiding properties, places a thermal stamp on the process. We see this also in situations far from the realm of black holes. For example, a detector tuned to the electromagnetic field and being accelerated uniformly in *empty* spacetime (described by the Minkowski metric) will measure a quantum noise characteristic of thermal electromagnetic radiation at a temperature proportional to its acceleration! The existence of this intriguing phenomenon was foreshadowed in work done[11] in 1975 by Paul C. W. Davies of King's College, London, and made theoretically compelling by Unruh[12] a year later. As Sciama has pointed out, one can again understand what is happening in terms of Hawking's pairs. Every physics student knows that to an accelerated observer part of the spacetime is invisible because he outruns all signals coming from those parts. For such an observer there is a horizon. Thus the elements are present for producing thermal radiation by a process analogous to that for a black hole. But here the energy is ultimately drawn from the agent which accelerates the detector.

Another example of thermal radiation ultimately associated with a horizon is that which all inertial observers see in the de Sitter universe, a cosmological model describing a classically empty universe with a high degree of symmetry, and which is separated into two regions by a horizon. Hawking and Gary Gibbons, who discovered this effect,[13] note that the area of that horizon has an entropy interpretation identical to that for a black hole. These examples show that the connection between gravitation and thermodynamics is likely to be a general feature of nature, relevant beyond the realm of black holes.

Stimulated emission

Hawking's analysis of black-hole radiance was an application of quantum-field theory in curved spacetime, a subject in which research has boomed since his pioneering work. These investigations have led to increasing understanding of the influence of gravitation on quantum processes, and of the quantum nature of gravitation. Yet the methods of this approach are complex, and the results do not lend themselves to description in simple terms. However, if one is only interested in understanding further the quantum–thermal properties of black holes, there is an alternative simpler approach, a direct outgrowth of black-hole thermodynamics, which makes use of general statistical arguments, and which may thus be called statistical black-hole physics. Its first accomplishment was the verification that

the generalized second law is respected in the face of Hawking's radiance, something I referred to earlier.

Statistical black hole physics has also resolved an intriguing paradox connected with black hole radiance. I mentioned how Zel'dovich predicted the existence of spontaneous emission by a Kerr hole in superradiant modes by interpreting superrradiance as stimulated emission, and insisting that spontaneous and stimulated emission go together. Now, according to Hawking there is also spontaneous emission in non-superradiant modes, such as, for example, all modes associated with the Schwarzschild hole. If stimulated emission is associated with it, why is there no superradiance in this case as in the previous one? Field theory did not clarify the situation because it did not seem to require the universal appearance of stimulated emission in every non-superradiant mode.[14]

A palatable resolution of the paradox emerged from a detailed statistical investigation of stimulated emission by black holes carried out by Amnon Meisels and myself at the Ben-Gurion University.[15] The central quantity in the analysis is $p(m|n)$, the probability that if n quanta in a given mode are incident on a black hole, m are returned outwards in that mode by *all* possible processes. We calculated $p(m|n)$ with the following assumptions:

▶ The black hole is in a thermal radiation bath
▶ The mean number of quanta the black hole returns under the influence of the thermal radiation incident on it is the sum of the spontaneous emission and a fraction $1 - \Gamma$ "reflected" from the incident mean number
▶ The probability distribution has a maximal entropy, for the same reasons I mentioned in connection with the heuristic explanation of Hawking's process. Prakash Panangaden and Robert Wald have shown[16] that this same probability can be derived from the field theory, albeit more laboriously.

Curiously, the probability $p(m|n)$ is not just the composite of the probability distributions for spontaneous emission, $p(m|0)$, and that for ordinary scattering of indistinguishable bosons. A third component distribution is needed to reproduce it. This was only isolated recently, and its form leaves no doubt that it describes stimulated emission in *every* mode: the mean number of quanta sent out by the corresponding process is proportional to n, as one would expect for stimulated emission. The proportionality coefficient is none other than Einstein's coefficient of stimulated emission, $B_\downarrow$.

As in the case of atomic transitions, $B_\downarrow$ turns out to equal the mean number of spontaneously emitted bosons, which is just the Einstein coefficient of spontaneous emission A. One can likewise calcu-

late the coefficient of absorption, $B_\uparrow$; it obeys the relation

$$\Gamma = B_\uparrow - B_\downarrow$$

We can now see that the classical absorption coefficient Γ is not just the quantum absorption coefficient, but is less than it by the stimulated emission coefficient (after all, stimulated emission suppresses scattering). The explicit forms of $B_\downarrow$ and $B_\uparrow$ make it clear that for a mode satisfying the superradiance condition, stimulated emission wins ($B_\downarrow > B_\uparrow$), so that the classical reflection coefficient, $1 - \Gamma$, is larger than unity. This is precisely what superradiance means! By contrast, if the mode does not satisfy the superradiance condition, absorption dominates stimulated emission ($B_\downarrow < B_\uparrow$), $1 - \gamma$ is less than unity, so that classically the hole absorbs, and there is no superradiance. Thus a Schwarzschild hole is capable of stimulated emission, but not of superradiance.

Counting interior configurations

In quantum physics the ratio of B-coefficients, $B_\uparrow/B_\downarrow$, equals the ratio of degeneracy factors of the upper to the lower energy level of the transition in question. For an equilibrium black hole, the ratio can easily be shown[16] to equal the ratio of the quantity we have called the number of possible interior configurations, $\exp(S_{bh}/k)$, evaluated for the hole's parameters before emission of a quantum in the mode in question, to the same number after emission. Thus if we regard the hole as a conventional quantum system, each of its states acts as if it were degenerate, with a multiplicity $\exp(S_{bh}/k)$. A more dramatic illustration of the interpretation of S_{bh} in terms of interior configurations would be hard to come by. The possible interior configurations act as if they were all present simultaneously in determining how the hole radiates. Alternatively, if one has confidence in the interpretation of S_{bh}, the result underlines the basic similarity of black holes to conventional quantum radiating systems.

Convincing as it is, the interpretation of S_{bh} has not yet been supplemented by a general method for explicitly counting interior configurations and demonstrating the connection with S_{bh}. Ulrich H. Gerlach has analyzed[17] a special model in which counting can be done schematically and has obtained rough agreement with S_{bh}. A different approach, which, though not formally a counting method, has great potential, has been advocated by Gibbons and Hawking[18] in the framework of a program for quantizing gravitation using Richard Feynman's "sum over histories" approach. Here the quantum amplitude for a system to evolve from an initial to a final state in time t is given by a certain phase factor summed over all conceivable evolutions between the two states. Gibbons and Hawking computed the amplitude for evolution of a geometry (gravitational field) that contains a black hole region, for complex values of the coordinates. Curiously, this geometry is periodic in imaginary time, it, with a period τ that is determined by M, Q, and L. Gibbons and Hawking were then able to exploit a trick well known to workers in statistical physics: the *formal* amplitude for a system to evolve back to its initial state in an *imaginary* time, it, equals its partition function (or statistical sum over states) for the (real) temperature h/kt. Because of its periodicity the hole's geometry evolves back to the initial state after "time" τ, and thus a thermal partition function is defined for it. The temperature turns out to be none other than T_{bh} (with $\eta = \frac{1}{4}$), and the entropy derived from the partition function is none other than S_{bh}. The agreement with the values implied by Hawking's radiance is proof that the thermal properties of the gravitational field are very real and manifest themselves in varied contexts.

Space limitations forbid a detailed survey of the applications and ramifications of the thermodynamics of black holes. Yet no description of the subject could be representative without mentioning some highlights. Hawking[6] has employed standard thermodynamic reasoning to deduce the criterion for a black hole to condense stably out of thermal radiation (when can droplets condense out of vapor?). Likewise, Davies[19] has shown that the charged Kerr holes are separated by the values of their parameters into two distinct classes connected by a second order phase transition like the λ point of He^4. The role of irreversibility in black-hole thermodynamics has been put in a new light by Philip Candelas and Sciama[20] who have demonstrated the close connection between quantum fluctuations of the gravitational field at the horizon and the dissipation reflected in the irreversibility, a connection analogous to that between noise and resistance in an electrical resistor.

These and other examples underline the basic similarity of the bizarre black holes to everyday objects in the quantum domain, a similarity which surfaces clearly in the thermodynamic viewpoint. Because of this the dream of understanding the quantum aspects of the gravitational field is closer to fulfillment today than a decade ago.

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